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In this paper, a system of nonlinear differential equations describing the signal transduction process in human neuroendocrine treatment of depression is considered. The model is based on the proposed by 

\section{Introduction}

\subsection{AMS (No Stop) Classification: 92D20}

\begin{abstract}
Periodic solution may be observed under appropriate conditions on the system parameters and in the problem setting. The dependence of the existence of solutions on the system parameters is shown. Finally, it is shown that the solutions are stable.
\end{abstract}

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\textbf{Depressant Drug Treatment}

\textbf{Transduction in Human Under Inductive Signal Transduction of Nonlinear Model of Signal}

\textbf{Dynamics and Applications} 19 (2010) 651-666
account the amplification effect that the secondary hormone exerts on the primary external signals. In [4], measurements of intracellular cAMP were made using Fisher rat thyroid cells expressing type II vasopressin receptors. This experimental data was then fitted with the cAMP level calculated from the model in [2], observing that the simulated curve fits the experimental data rather well although there are some discrepancies that need further investigations. It also suggests that other physical factors may be at play which we need to take into account.

Abnormalities of signal transduction pathways have been linked to the development of many serious disorders, such as cancer which derives from a cell that has lost the ability to respond normally to controls from outside, or inside, the cell [5]. Many tumors produce ectopic amounts of biologically active hormones that create dysfunctions of the signal transduction process leading to abnormal effects. Hormones and antihormones are used to treat certain types of cancer. Many cancers are related to the status of hormones in the body. An avenue of cancer treatment is to utilize appropriate hormones as chemotherapeutic agents. For example, tamoxifen can interfere with the offensive effects of estrogen, resulting in the inhibition of cellular growth of the tumor. For another example, Vasopressin has been proposed for its potential effect of slowing down the flow of blood that tumors depend on for growth [6].

We incorporate such periodic drug treatments or external signals by using the following impulsive system.

\[
\begin{align*}
\frac{dx_1}{dt} &= -a_1 x_1 - \frac{b_1 x_1}{b_2 + x_1^2} + \frac{b_3 x_1}{b_4 x_1 + x_2} = f_1(x_1, x_2), \quad t \neq kT \\
\frac{dx_2}{dt} &= -a_2 x_2 + a_3 x_1 = f_2(x_1, x_2), \quad t \neq kT \\
\Delta x_1 &= -px_1, \\
\Delta x_2 &= \mu \\

t &= kT, \\
\end{align*}
\]

where \( k \in \mathbb{Z}_+ \), where

\[
\Delta x_i(t) = x_i(t^+) - x_i(t), \quad i = 1, 2,
\]

\( T \) is the period of the impulsive effect of drug treatments, \( x_1(t) \) is the density above the basal level of ligand coupled receptors (LCR) on the cell membrane, \( x_2(t) \) is that of the inhibiting agent, \( a_1 \) is the specific removal rate of \( x_1 \) by natural means, \( a_2 \) is the specific removal rate of \( x_2 \) by natural means, and \( a_3 \) is the rate of production of \( x_2 \) per unit of the hormone coupled receptors \( x_1 \).

The second term on the right of equation (1.1) accounts for the internalization of \( x_2 \) across the cell membrane which is assumed here to saturate as \( x_1 \) becomes high.

The third term accounts for the amplification effect of the secondary hormone on the first messenger’s signaling strength. In [1], this effect was assumed to vary directly as the level of the secondary hormone \( C(t) \) at any time \( t \), the production rate of which was assumed to vary as the square of the activated units of adenylate cyclase of cAMP. The activated units of inhibiting agents \( l \) as:

\[
\begin{align*}
\frac{dx_1}{dt} &= -a_1 x_1 - \frac{b_1 x_1}{b_2 + x_1^2} + \frac{b_3 x_1}{b_4 x_1 + x_2} = f_1(x_1, x_2), \quad t \neq kT \\
\frac{dx_2}{dt} &= -a_2 x_2 + a_3 x_1 = f_2(x_1, x_2), \quad t \neq kT \\
\Delta x_1 &= -px_1, \\
\Delta x_2 &= \mu \\

t &= kT, \\
\end{align*}
\]

so that, according to [1] and [2], in the form

\[
C(t) = b_5 - k_6 b_6. \quad \text{Details of the derivation}.
\]

In this paper, the primary hormone \( x_3 \) of LCR above the basal level, is considered that it may be more realistic to vary as the current level of the hormone \( x_3 \), to arrive at

\[
\Delta x_3 = \nu x_3 \quad t = kT,
\]

in which we have also assumed that the hormone (cAMP) is negligible at time \( 0 \). This then leads to the third term of the constant of variation.

Equation (1.3) accounts for the stimulating strength in LCR by the fraction \( p \), \( 0 < p < 1 \), for the hormone \( \mu, \mu > 0 \).

In Section 2, we give some lemmas. In Section 3, the conditions which lead to a periodic solution at the vanishing level are possible provided the treatment time and stability of positive periodic solution is shown. Section 4. The last section then
Section 4. The last section then contains numerical results and concluding remarks.

Section 5. The last section also contains numerical results and concluding remarks.

Section 6. The last section further contains numerical results and concluding remarks.

Section 7. The last section concludes with a discussion of the results obtained in this paper.
2. PRELIMINARIES

In order to prove our main results, we need to give some lemmas which need the following definition [7].

**Definition 2.1.** Let $V : \mathbb{R}_+ \times \mathbb{R}_+^2 \to \mathbb{R}$, where $\mathbb{R}_+ = [0, \infty)$, be continuous in $(nT, (n+1)T] \times \mathbb{R}_+^2$ and for each $x \in \mathbb{R}_+^2$, $n \in \mathbb{Z}_+$, \(\lim_{(t,y) \to (nT^+,x)} V(t,y) = V(nT^+,x)\) exists. Also, let $V$ be locally Lipschitzian in $x$. Then, for $(t,x) \in (nT, (n+1)T] \times \mathbb{R}_+^2$, the upper right derivative of $V(t,x)$ with respect to the impulsive differential system (1.1)-(1.3) is defined as

$$D^+V(t,x) = \lim_{h \to 0^+} \frac{1}{h} [V(t+h,x+hf(t,x)) - V(t,x)],$$

where $f = (f_1, f_2)$.

The solution $x(t) = (x_1(t), x_2(t))$ of (1.1)-(1.3) is a piecewise continuous function, $x : \mathbb{R}_+ \to \mathbb{R}_+^2$, continuous on $(nT, (n+1)T)$, $n \in \mathbb{Z}_+$, and $x(nT^+) = \lim_{t \to nT^+} x(t)$ exists. Thus, the global existence and uniqueness of solutions of (1.1)-(1.3) are assured by the smoothness properties of $f$.

Since $\frac{dx_1}{dt} = 0$ whenever $x_1(t) = 0$, $t \neq nT$, $\frac{dx_2}{dt} > 0$ whenever $x_2(t) = 0$, $t \neq nT$, and $x_1(nT^+) = (1-p)x_1(nT)$, $0 < p < 1$, $x_2(nT^+) = x_2(nT) + \mu$, $\mu > 0$, we have the following lemma.

**Lemma 2.2.** Suppose $x(t) = (x_1(t), x_2(t))$ is a solution of (1.1)-(1.3) with $x_i(0^+) \geq 0$, $i = 1, 2$. Then, $x_i(t) > 0$, $i = 1, 2$, for $t \geq 0$ if $x_i(0^+) > 0$, $i = 1, 2$.

Next, we show that all solutions of (1.1)-(1.3) are uniformly ultimately bounded.

**Lemma 2.3.** There exists a constant $M > 0$ such that $x_i \leq M$, $i = 1, 2$, for each solution $x(t) = (x_1, x_2)$ of (1.1)-(1.3) with all $t$ sufficiently large if

$$a_1 > a_3$$

(2.1)

**Proof.** Letting $V(t) = V(t, x(t)) = x_1(t) + x_2(t)$, and choosing

$$c = \min(a_1 - a_3, a_2)$$

which is positive, we have when $t \neq kT$ that

$$D^+V(t) + cV = -a_1x_1 - \frac{b_1x_1}{b_2 + x_1^2} + \frac{b_3x_1}{b_4x_1 + x_2} - a_2x_2 + a_3x_1 + cx_1 + cx_2$$

$$\leq (-a_1 + c + a_3)x_1 + b + (-a_2 + c)x_2 \leq b$$

where $b = \frac{b_3}{a_3}$. That is, when $t \neq kT$, $D^+V \leq -cv + b$.

When $t = t_k = kT$,

$$V(kT^+) = x_1(kT^+) + x_2(kT^+) = x_1(t_k) - px_1 + x_2(t_k) + \mu \leq V(t_k) + \mu$$

(2.5)

for $kT < t \leq (k + 1)T$, and $\bar{x}_2(t)$.

Finally, we consider the following differential equation

$$\frac{dx_2}{dt} = x_2(kT^+) - \frac{b}{c} \frac{\mu e^{-\frac{t}{c}}}{e^{\frac{t}{c}} - 1}$$

(2.2)

$$x_2(kT^+)$$

(2.3)

We see that the following function

$$\bar{x}_2(t)$$

(2.4)

for $t \in (kT, (k + 1)T]$, $k \in \mathbb{Z}_+$, is a solution of (2.2)-(2.4). Thus, the solution of (2.2)-(2.4) is uniformly ultimately bounded on $[0, \infty)$ and therefore we have the following lemma.

**Lemma 2.4.** The system (2.2) has every solution $x_2(t)$ of (2.2)-(2.4).

Hence, system (1.1)-(1.3) has

$$x(t)$$

(2.5)

for $kT < t \leq (k + 1)T$, and $\bar{x}_2(t)$. Thus, system (1.1)-(1.3) has
\[ t + (\phi_t)A \leq (t + \phi_t)x + t\phi_z \quad \text{for } t > 0 \]
\[ q + z \leq x \in [0, \infty) \]
\[ \frac{\phi_z}{\phi_z} + \frac{t \phi_z}{\phi_z} + \frac{t \phi_z}{\phi_z} + \frac{t \phi_z}{\phi_z} \]

\[ \lim_{t \to \infty} \frac{q + z}{\phi_z} = 0 \]

By Lemma 2.2 in [5] for \( t > 0 \), we have \([L(1 + \gamma), L] \) is uniformly bounded. Hence, by the definition of \( A \), there is an \( \tilde{A} \) such that all \( \tilde{A} \) is uniformly ultimately bounded. Then, the following system is continuous and \( (x, t) \to \tilde{A} \) is a piecewise continuous function.
3. VANISHING STIMULUS AND PERMANENCE

We first give the conditions that guarantee the locally asymptotic stability of the periodic solution \((0, \bar{x}_2(t))\) at the point of vanishing stimulus.

**Theorem 3.1.** Let \(x(t)\) be any solution of (1.1)–(1.3). Then, \((0, \bar{x}_2(t))\) is locally asymptotically stable if

\[ T < T_{\text{max}} \]

with

\[ \frac{4\mu b_3}{a_2} \sinh^2 \frac{a_2 T_{\text{max}}}{2} = \left( a_1 + \frac{b_1}{b_2} \right) T_{\text{max}} + \ln \frac{1}{1 - p} \]

**Proof.** Consider a small amplitude perturbation of \((0, \bar{x}_2(t)):\)

\[ x_1(t) = u(t) \]
\[ x_2(t) = \bar{x}_2 + v(t) \]

We may write

\[ \begin{pmatrix} u(t) \\ v(t) \end{pmatrix} = \Phi(t) \begin{pmatrix} u(0) \\ v(0) \end{pmatrix}, \quad 0 < t < T \]

where \(\Phi\) satisfies

\[ \frac{d\Phi}{dt} = \begin{pmatrix} -a_1 - \frac{b_1}{b_2} + \frac{b_3}{\bar{x}_2} & 0 \\ a_3 & -a_2 \end{pmatrix} \Phi \]

and \(\Phi(0) = I\), the identity matrix. Hence, the fundamental solution matrix is

\[ \Phi = \begin{pmatrix} \exp \int_0^t \left( a_1 + \frac{b_1}{b_2} - \frac{b_3}{\bar{x}_2} \right) ds & 0 \\ 0 & \exp \int_0^t (-a_2) ds \end{pmatrix} \]

for which it is not necessary to find the exact expression for (*) since it is not required in the following analysis.

Linearization of (1.3) gives

\[ \begin{pmatrix} u(kT^+) \\ v(kT^+) \end{pmatrix} = \begin{pmatrix} 1 - p & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u(kT) \\ v(kT) \end{pmatrix} \]

The stability of the periodic solution \((0, \bar{x}_2(t))\) is determined by the eigenvalues of

\[ M_0 = \begin{pmatrix} 1 - p & 0 \\ 0 & 1 \end{pmatrix} \Phi(T) \]

which are

\[ v_1 = (1 - p)e^{\int_0^T \left( -a_1 - \frac{b_1}{b_2} + \frac{b_3}{\bar{x}_2} \right) ds} \]

and

\[ v_2 = e^{-a_2 T} < 1 \]

According to the Floquet theory, we observe that

\[ \int_0^T \frac{1}{\bar{x}_2} ds < \frac{4\mu b_3}{a_2} \sinh^2 \frac{a_2 T_{\text{max}}}{2} \]

Hence, \(|v_1| < 1\) if

\[ b_3 \cdot \frac{4\mu}{a_2} \sinh^2 \frac{a_2 T_{\text{max}}}{2} \]

Letting \(\Omega_1(T)\) be the function on the left-hand side, then we see that \(\Omega_1(0) - \Omega_2(0) < 0\), \(\Omega_1 - \Omega_2\) is increasing for \(T > 0\), and hence \(\Omega_1(T_{\text{max}}) = \Omega_2(T_{\text{max}})\) at which \(\Omega_1(T_{\text{max}}) = \Omega_2(T_{\text{max}})\) is complete.

We next investigate the permanence property definition.

**Definition 3.2.** System (1.1)–(1.3) is said to be permanence if \(m, M > 0\) (independent of initial values) and \(x(t)\) with all initial values \(x_i(0^+) > 0\) exist and may depend on the initial values.

**Theorem 3.3.** The system (1.1)–(1.3) is permanent.

**Proof.** Suppose \(x(t) = (x_1, x_2)\) is a solution of (1.1)–(1.3) with all initial values \(x_i(0^+) > 0\) then, from Lemma 2.2, there is an \(M > 0\) such that

\[ \frac{dx_2}{dt} \leq \frac{dx_2}{dt} < x_2(t^+) \]

and we have

\[ x_2(t^+) \leq x_2(t) \leq x_2(t) \]

for all \(t \geq 0\).
\[ s - (1)x < (1)x \]

and we have

\[ L^\alpha = 1, \quad (s + (1)x = (s + 1)x \]

\[ L^\alpha \neq 1, \quad \frac{\alpha x}{\exp \alpha} < \frac{\alpha p}{\exp \alpha} \]

From (1), we know

Lemma 3. There is a solution for \( t = 1 \) in \( (a, 1) \) \( \exists x(t, \theta) \) such that for large enough \( t \), we have

\[ x(\alpha) < L \]

Suppose \( \Phi \) is a solution of the equation of \( (a, 1) \) \( - (1, 1) \)

Definition 3. The system is permanent if it holds and

\[ x\alpha \beta L < L \]

may depend on the initial value if is permanent if it holds.

Theorem 3. The system is permanent if by first giving the following definition.

We next investigate the permanence of \( (a, 1) \) - \( (1, 1) \).

Complete:

The proof is similar to the proof for \( (a, 1) \) \( - (a, 1) \) \( \exists x(\alpha, \beta) \) such that the function on the left of \( (a, 1) \) \( - (a, 1) \)

Letting \( (a, 1) = \text{function on the left of } (a, 1) \)

\[ \frac{d}{dt} x(t) + L \left( t_0 + \frac{t}{t_0} \right) > \frac{\alpha}{\exp \alpha} x(t) \]

Hence, \( t > 1 / \alpha \), then

\[ (a, 1)^{\alpha} x(t) \leq \frac{\alpha t_0}{\exp \alpha} \]

\[ (a, 1)^{\alpha} x(t) \leq \frac{\alpha t_0}{\exp \alpha} \]

\[ (1 - \alpha t_0)^{\alpha} (a, 1)^{\alpha} - 1 = \alpha \exp \alpha \]

Note that (1) is locally stable if \( \alpha > 1 \).

According to the Proof Theory, \( (a, 1)^{\alpha} x(t, \theta) \) is locally stable if \( \alpha > 1 \).

AND PERMANENCE

AND STABILITY

AND TRANSITE
for all $t$ large enough and some $e > 0$, so that

$$x_2(t) \geq \frac{\mu e^{-a_2 T}}{1 - e^{-a_2 T}} - e \equiv m_2$$

for $t$ large enough. Thus, we only need to find an $m_1 > 0$ such that

$$x_1(t) \geq m_1, \quad \text{for } t \text{ large enough.}$$

**Step 1** From the arguments in Theorem 3.1, we see that if $T > T_{max}$, then

$$\exp \int_{kT}^{T_{max}} \left( -a_1 - \frac{b_1}{b_2} + \frac{b_3}{b_2} \right) dt > 1$$

where

$$\hat{x}_2 = \frac{\mu \exp(-a_2(t - kT))}{1 - \exp(-a_2 T)}$$

By continuity of the integral in (3.6), if $m_3 > 0$ and $e_1 > 0$ are small enough, then

$$\exp \int_{kT}^{T_{max}} \left( -a_1 - \frac{b_1}{b_2} + \frac{b_3}{b_4 m_3 + \hat{z} + e_1} \right) dt > 1$$

also, where $\hat{z} = \hat{x}_2 + \frac{a m_3}{a_2}$.

We will prove that $x_1(t) < m_3$ cannot hold for all $t \geq 0$. Otherwise,

$$\frac{dx_2}{dt} = -a_2 x_2 + a_3 x_1 \leq -a_2 x_2 + a_3 m_3, \quad t \neq kT$$

$$x_2(t^+) = x_2(t) + \mu, \quad t = kT$$

if $x_1(t) \geq 0$

We then obtain $x_2(t) \leq z(t)$ and $z(t) \to \hat{z}(t)$, $t \to \infty$, where $z(t)$ is the solution of

$$\begin{cases}
\frac{dz}{dt} = -a_2 z(t) + a_3 m_3, & t \neq kT \\
z(t^+) = z(t) + \mu, & t \neq kT \\
z(0^+) = x_2(0^+) 
\end{cases}$$

and

$$\hat{z}(t) = \frac{\mu \exp(-a_2(t - kT))}{1 - \exp(-a_2 T)} + \frac{a_3}{a_2} m_3, \quad t \in (kT, (k + 1)T]$$

Therefore, there exists a $t_1 > 0$ such that

$$x_2(t) < z(t) < \hat{z}(t) + e_1$$

and

$$\frac{dx_1}{dt} \geq x_1(t) \left( -a_1 - \frac{b_1}{b_2} + \frac{b_3}{b_4 m_3 + \hat{z} + e_1} \right), \quad t \neq kT$$

$$x_1(t^+) = (1 - p)x_1(t), \quad t \neq kT$$

for $t \geq t_1$. Let $N \in \mathbb{Z}_+$ and $NT \geq t_1$.

**Case 2.1** $t^* = k_1 T$, for some $k_1 \in \mathbb{Z}_+$

and

$$m_3 > x_1(t^*)$$

Choose $k_2, k_3 \in \mathbb{Z}_+$ such that

$$(1-p)^{k_2} \exp(k_2 \eta_1 T) \eta_1$$

where $\eta_1 = -a_1 - \frac{b_1}{b_2} + \frac{b_3}{M_0 (1+\alpha)} < 0$.

Let $T' = k_2 T + k_3 T$. We claim

Otherwise (3.9) holds for $t^* + k_2 T$,

$$x_1(t^*)$$

On the other hand, for $t \in [t^*, t^* + k_2 T)$

$$\frac{dx_1}{dt} \geq x_1(t) \left( -a_1 - \frac{b_1}{b_2} + \frac{b_3}{b_4 m_3 + \hat{z} + e_1} \right), \quad t \neq kT$$

$$x_1(t^+) = (1 - p)x_1(t), \quad t \neq kT$$

Integrating (3.11) over $[t^*, t^* + k_2 T)$,

$$x_1(t^* + k_2 T) > x_1(t^*)$$

Substituting into (3.10), we have

$$x_1(t^* + T') \geq \eta$$

**Step 2** If $x_1(t) \geq m_3$, for all $t > t_0$ such that

Then, let $t^* = \inf \{ t : x_1(t) < m_3 \}$

Increasing (3.9) over $(kT, (k + 1)T)$,

$$x_1((k + 1)T) \geq x_1(kT) (1 - p) \exp \int_{kT}^{(k + 1)T} \left( -a_1 - \frac{b_1}{b_2} + \frac{b_3}{b_4 m_3 + \hat{z} + e_1} \right) dt > 1$$

which is a contradiction to the bound on $x_1(t)$.

Therefore, we must have $x_1(t) < m_3$ for all $t \geq 0$.
\[ w < \varepsilon \mu (L_y \psi) dy \exp \varepsilon_y (d - 1)^{w} \leq (L + I)^{1/2} x \]

Substituting into (3.10), we have

\[ (L_y \psi) dy \exp \varepsilon_y (d - 1)^{w} \leq (L_y^2 + I)^{1/2} x \]

Integrating (3.11) over \([J, I + \varepsilon, I]\), we have

\[ \int_{J}^{I + \varepsilon} \left( \frac{t + \varepsilon}{t} \right) dt \leq \int_{J}^{I + \varepsilon} \frac{dp}{|p|} \]

On the other hand, for \(t \in [J, I + \varepsilon, I]\), we have from (3.9)

\[ c_w \mu (L_y^2 + I)^{1/2} x \leq (L + I + \varepsilon)^{1/2} x \]

Otherwise, (3.9) holds for \(I + \varepsilon \geq I \geq L_y^2 + I \geq y \geq w \).

So as in Step 1, we have

\[ w < (\varepsilon_I)^{1/2} x \]

We claim that there must be \(y \in [J, I + \varepsilon, I]\) such that

\[ \frac{(t + \varepsilon/2)}{t} \geq \frac{e^0}{I} \quad \text{for all } t \in [J, I + \varepsilon, I]. \]

Choose \(k \in \mathbb{Z}^+\) such that

\[ (d - 1)^{w} \leq (\varepsilon_I)^{1/2} (d - 1) < (\varepsilon_I)^{1/2} \]

and

\[ \{w > (\varepsilon_I)^{1/2} \} \text{ holds for all } t \text{ such that } w > (\varepsilon_I)^{1/2} \]

Then, let \(J = \varepsilon_I^{1/2} \in [J, I + \varepsilon, I]. \)

Then, there are two possible cases for \(t \in [J, I + \varepsilon, I]. \)

Case 2. \(I 0 \text{ or } \varepsilon 0 \text{ or } \varepsilon \leq \varepsilon_I\). Then, there is a contradiction to the boundedness of \(x(t)\).

Hence, there is a such that

\[ w \leq (\varepsilon_I)^{1/2} \]

\[ \varepsilon_I^{1/2} \leq (\varepsilon_I)^{1/2} \]

that

which is a contradiction to the boundedness of \(x(t)\).

Hence, there is a such that

\[ w \leq (\varepsilon_I)^{1/2} \]

Then

\[ \varepsilon_I^{1/2} \leq (\varepsilon_I)^{1/2} \]

that

\[ w \leq (\varepsilon_I)^{1/2} \]

Therefore, we have

\[ \mu (L_y) \leq \left( \frac{1 + \varepsilon}{\varepsilon} + \frac{w}{\varepsilon} \right) \left( \frac{1 + \varepsilon}{\varepsilon} - 1 \varepsilon \right) \]

\[ \int_{J}^{I + \varepsilon} \left( \frac{1}{\varepsilon} \right) dy \exp (d - 1) (L_y) \leq (L_y + I)^{1/2} x \]

Integrating (6.3) over \([J, I + \varepsilon, I]\), we have

\[ \int_{J}^{I + \varepsilon} \left( \frac{1}{\varepsilon} \right) dy \exp (d - 1) (L_y) \leq (L_y + I)^{1/2} x \]

Thus, we have

\[ \mu (L_y) \leq \left( \frac{1 + \varepsilon}{\varepsilon} + \frac{w}{\varepsilon} \right) \left( \frac{1 + \varepsilon}{\varepsilon} - 1 \varepsilon \right) \]

\[ \int_{J}^{I + \varepsilon} \left( \frac{1}{\varepsilon} \right) dy \exp (d - 1) (L_y) \leq (L_y + I)^{1/2} x \]
which is a contradiction.

Hence, there is a \( t_2 \in (t^*, T'] \) such that

\[
x_1(t_2) > m_3
\]

So, let \( \tilde{t} = \inf_{t \leq t'} \{ t : x_1(t) > m_3 \} \). Then, for \( t \in (t^*, \tilde{t}) \), \( x_1(t) \leq m_3 \) and \( x_1(\tilde{t}) = m_3 \) since \( x_1(t) \) is left continuous and

\[
x_1(t^*) = (1 - p)x_1(t) \leq x_1(t)
\]

when \( t = kT \).

For \( t \in (t^*, \tilde{t}) \) suppose \( t \in (t^* + (l - 1)T, t^* + lT], l \in \mathbb{Z}_+ \) and \( l \leq k_2 + k_3 \). From (3.10), we have

\[
x_1(t) \geq x_1(t^*) (1 - p)^{l-1} \exp((l - 1)\eta T) \exp(\eta (t - (t^* + (l - 1)T))
\]

\[
\geq m_3 (1 - p)^l \exp(\eta T)
\]

\[
\geq m_3 (1 - p)^{k_2 + k_3} \exp((k_2 + k_3)\eta T) \equiv m'_3
\]

So, we have \( x_1(t) \geq m'_3 \) for \( t \in (t^*, \tilde{t}) \) and \( x_1(\tilde{t}) \geq m_3 \). We can repeat the argument for \( t > \tilde{t} \) to obtain the result that \( x_1(t) \geq m_1 > 0 \) for \( t \) large enough.

Case 2.2 \( t^* \neq kT \) for all \( k \in \mathbb{Z}_+ \). Then,

\[
x_1(t) \geq m_3 \quad \text{for} \ t \in (t_1, t^*)
\]

and

\[
x_1(t^*) = m_3.
\]

Suppose \( t^* \in (k'_1 T, (k'_1 + 1)T) \) for some \( k'_1 \in \mathbb{Z}_+ \). There are 2 possible cases for \( t \in (t^*, (k'_1 + 1)T) \).

Case 2.2 a) \( x_1(t) \leq m_3 \) for all \( t \in (t^*, (k'_1 + 1)T) \). We claim that there must be a \( t'_2 \in [(n'_1 + 1)T, (n'_1 + 1)T + T] \) such that \( x_1(t'_2) > m_3 \). Otherwise, similarly to Case 2.1, we get

\[
x_1((k'_1 + 1 + k_2 + k_3)T) \geq x_1((k'_1 + 1 + k_2 + k_3)T) \eta^{n_3}
\]

On the other hand, for \( t \in (t^*, (k'_1 + 1)T) \), (3.11) holds on \([t^*, (k'_1 + 1 + k_2 + k_3)T]\), and \( x_1(t) \leq m_3 \), so that we have

\[
x_1((k'_1 + 1 + k_2)T) \geq m_3 (1 - p)^{k_2} \exp((k_2 + 1)\eta T)
\]

Thus,

\[
x_1((k'_1 + 1 + k_2 + k_3)T) \geq m_3 (1 - p)^{k_2} \exp((k_2 + 1)\eta T) \eta^{n_3} > m_3,
\]

a contradiction.

Let \( \tilde{t} = \inf_{t \leq t'} \{ t : x_1(t) > m_3 \} \). Then,

\[
x_1(t) \leq m_3 \quad \text{for} \ t \in (t^*, \tilde{t})
\]

and

For \( t \in (t^*, \tilde{t}) \), suppose \( t \in (k'_1 T + (l')T \).

Then, we have

\[
x_1(t) \geq m_3 (1 - p)^l \exp(\eta T)
\]

\[
\geq m_3 (1 - p)^l
\]

So, \( x_1(t) \geq m_3 \) for \( t \in (t^*, \tilde{t}) \). The \( x_1(\tilde{t}) \geq m_3 \). We thus get \( x_1(t) \geq m_3 \).

Case 2.2 b) There exists a \( t \in (t^*, \tilde{t}) \)

Let \( \tilde{t} = \inf_{t \leq t'} \{ t : x_1(t) > m_3 \} \). Then,

\[
x_1(t) \geq m_3
\]

and

For \( t \in (t^*, \tilde{t}) \), (3.11) holds and integrating

\[
x_1(t) \geq x_1(t^*) \exp((t - t^*)\eta T)
\]

Using the fact that \( x_1(\tilde{t}) \geq m_3 \), we have

Hence, we obtain \( x_1(t) \geq m_1 > 0 \).

We now investigate the possibility of the system (1.1)-(1.3) near \( (0, \tilde{x}_2, 0) \) (3.11) holds on \([t^*, (k'_1 + 1 + k_2 + k_3)T]\).

For this purpose, it is more convenient as given in (3.2). The system (1.1)

\[
\frac{dx_1}{dt} = -a_2 x_1 + \frac{dz_1}{dt} = -a_2 z_1
\]

\[
\frac{dz_2}{dt} = -a_1 x_2 - \frac{dz_2}{dt} = -a_1 z_2
\]

\[
\frac{dz_1}{dt} = \frac{dx_1}{dt} - \frac{dz_2}{dt} = \frac{dx_1}{dt} - \frac{dz_2}{dt}
\]

\[
\frac{dx_1}{dt} = \frac{dx_1}{dt} - \frac{dz_2}{dt}
\]

By Theorem 2 of [8], we then have

\[\text{Theorem 3.4. The system (1.1)-(1.3) is locally exponential stable at the critical point (0, \tilde{x}_2, 0) if (2.1) and (3.5) hold.}\]
Theorem 3.2. The system (1.1)-(1.3) of (5.3)-2.1.9 has a positive periodic solution which is stable.

By Theorem 2.1.7 we then have the following result.

Let\( Y = \{ (i)^{x}x^{d} = (i)^{y}x^{d} \mid \}
\]

\[\begin{align*}
\text{if } Y \neq 1 & \quad \frac{1}{r} + \frac{a}{r} - \frac{r}{r} = \frac{a}{p} \\
\text{if } Y \neq 1 & \quad \frac{a}{r} + \frac{a}{r} - \frac{r}{r} = \frac{a}{p}
\end{align*}\]

As given in (5.2) the system (1.1)-(1.3) is not now within as the purpose of it more convenient to exchange + and 0 and let t = 0. Hence, we obtain in the region (1.1) and the proof is complete.

Let the last term \( x^{\nu} \) be present and apply the above argument again to \( t \), we have

\[\begin{align*}
\text{with } (i)^{x}x^{d} \leq (i)^{y}x^{d} \\
\text{for } t \in (1.1) \text{ holds and interchanging (1.1), (1.1)} \text{ we have}
\end{align*}\]

\[\begin{align*}
\text{and } (i, t) \in E \quad \text{for } t < (i)^{x}
\end{align*}\]

Let \( \{ (i)^{x}x^{d} \} \) then the region (1.1) is such that \( (i)^{y}x^{d} \) exists and \( a \neq (i)^{x}x^{d} \).

Case 2: \( x^{\nu} \geq 0 \). If \( a \neq (i)^{x}x^{d} \) then we have

\[\begin{align*}
\text{and } (i, t) \in E \quad \text{for } t < (i)^{x}
\end{align*}\]
Proof. Relying on the notations used in [8], we have
\[ F_1(x_1, x_2) \equiv -a_2x_1 + a_3x_2 \]
\[ F_2(x_1, x_2) \equiv -a_2x_2 - \frac{b_1x_2}{b_2 + x_2^2} + \frac{b_3x_2}{b_4x_2 + x_1} \]
\[ \Theta_1(x_1, x_2) \equiv x_1 + \mu, \]
\[ \Theta_2(x_1, x_2) \equiv (1 - p)x_2 \]
\[ \zeta(t) \equiv (x_2(t), 0)^T \]
\[ x_0 \equiv (x_2(0), 0)^T. \]

We then can determine the relevant quantities as follows.
\[
\frac{\partial^2 \Phi_2}{\partial x_1 \partial x_2} (\tau_0, x_0) = \int_0^{\tau_0} \exp \left( \int_u^\tau \frac{\partial F_2}{\partial x_2} \exp \left( \int_u^u \frac{\partial F_2}{\partial x_2} \, ds \right) \, du \right) \, \left( \frac{\partial^2 F_2}{\partial x_1 \partial x_2} \right)_{(\tau_0, x_0)} < 0
\]

since \( \frac{\partial^2 \Phi_2}{\partial x_1 \partial x_2} (\tau_0, x_0) = \frac{-a_2}{x_2^2} < 0 \).

Since \( \frac{\partial^2 \Phi_2}{\partial x_1 \partial x_2} = 0 \), we have
\[ B = -\frac{\partial \Theta_2}{\partial x_2} \left( \frac{\partial^2 \Phi_2}{\partial \tau \partial x_2} + \frac{\partial \Phi_2}{\partial x_1} \frac{1}{a_0} \frac{\partial \Theta_1}{\partial \tau} \frac{\partial \Phi_1}{\partial x_1} \right)_{(\tau_0, x_0)} \]

Noting that, if (3.5) holds,
\[ a_0' = 1 - \frac{\partial \Theta_1}{\partial x_1} - \frac{\partial \Phi_1}{\partial x_1} \]
\[ \frac{\partial \Phi_1}{\partial x_1} \] \( (\tau_0, x_0) \) \( = \exp \int_0^{\tau_0} \frac{\partial F_1}{\partial x_1} \, ds \) \( (\tau_0, x_0) \) \( > 0 \)
\[ \frac{\partial \Phi_2}{\partial \tau} \] \( (\tau_0, x_0) \) \( = \frac{a_2 \mu \exp (-a_2 \tau)}{1 - \exp (-a_2 \tau)} \) \( < 0 \)
\[ \frac{\partial^2 \Phi_2}{\partial \tau \partial x_2} \] \( (\tau_0, x_0) \) \( = -\frac{\partial F_2}{\partial x_2} \exp \left( \int_0^{\tau_0} \frac{\partial F_2}{\partial x_2} \, ds \right) \) \( (\tau_0, x_0) \) \( > 0 \).

We conclude that
\[ B < 0 \]

Next, since \( \Theta_1 \) and \( \Theta_2 \) are linear we have [8]
\[ C = \frac{\partial \Theta_2}{\partial x_2} \left( \frac{b_0}{a_0} \frac{\partial^2 \Phi_2}{\partial x_2} - \frac{\partial \Phi_2}{\partial x_2} \right) \] \( (\tau_0, x_0) \)

Referring to [8] for the definitions of the partial derivative terms appearing above, we specifically have
\[ b_0' = -\left( \frac{\partial \Theta_1}{\partial x_1} \frac{\partial \Phi_1}{\partial x_2} + \frac{\partial \Theta_2}{\partial x_2} \frac{\partial \Phi_2}{\partial x_2} \right) \] \( (\tau_0, x_0) \) \( < 0 \)

so that \( d_0' = 0 \) at \( T = T_{\text{max}} \). That is, by Theorem 3.4, the system has a positive periodic solution having period of \( T = 130 \) units of time.

4. DISCUSSION

In Figure 1, a numerical simulation is seen to tend toward the perfect periodic solution containing impulses periodically. The corresponding Figure 2. Figure 3 shows a number of conditions in Theorem 3.4 holds that a periodic solution containing impulses having period of \( T = 130 \) units of time. The variable oscillations are seen in Figure 4.

Our analysis suggests a very small adjustment of the frequency \( \frac{1}{T} \) of values of \( p \) and \( \mu \), in order to obtain the conclusions indicate that we may be able to maintain even at the vanishing level of the impulses. On the other hand, if we choose a convenient fixed level, then it is shown so that \( T_{\text{max}} \), solved from equation (3.5), whichever case is the desirable one.
4. DISCUSSION AND CONCLUSION

Let \( \xi = \sum_{n=1}^{\infty} \frac{1}{n^2} \), then we have a periodic solution which is supported by the periodic solution. The system is stable if and only if \( \sum_{n=1}^{\infty} \frac{1}{n^2} < \frac{1}{2} \). 

Also, if \( \xi \geq 1 \), then \( \xi < 0 \).

Hence,

\[
0 > 0
\]

We are therefore led to

\[
0 < \int_{\xi}^{\xi + \xi q} \int dx = \frac{\xi}{\xi q}
\]

and

\[
0 > \left[ \int_{\xi}^{\xi + \xi q} \int dx \right] \left[ \frac{\xi}{\xi q} \right] = \frac{\xi}{\xi q}
\]
Thus, our work is expected to form a valuable basis for further investigations into how we could better manage and control such a complex signaling system, the proper function of which is crucially connected to human’s health and disease.

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Figure 1. Numerical simulation results showing that \( T < T_{\text{max}} \), showing a stable steady state in the control plane, exhibiting sustained fluctuations. Here, \( a_1 = 0.7, \ldots, b_5 = 119.6046 \).

Figure 2. The time series data corresponding to the case seen in Figure 1.
To the case seen in Figure 1.

**Figure 2.** The time series of $x_1$ in (a), and $x_2$ in (b), corresponding

$\max$ and $J_{\text{max}}$.

Here, $a_1 = 0.7, b = 0.2, c = 0.05, d = 0.1$. The phase plane exhibiting sustained oscillation in $x_1$ at vanishing level of $x_2$. Figure 2 shows the solution trajectory of the system (1.1)–(1.2) in the case.

**Figure 1.** Numerical solution of the system (1.1)–(1.2) in the case.

-0.05 0.0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5

-0.05 0.0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5

**Dynamics of a Nonlinear Model**
FIGURE 3. Numerical solution of the system (1.1)–(1.2) in the case that $T > T_{\text{max}}$, showing the solution trajectory tending toward a positive periodic solution with impulsive jumps. Here, $a_1 = 0.7, b_5 = 0.5, \mu = 1, p = 0.3, T = 130$, and $T_{\text{max}} = 119.6046$.

FIGURE 4. The time series of $x_1$, in 4a), and $x_2$, in 4b), corresponding to the case seen in Figure 3.

Dynamic Systems and Applications

ON THE $\omega$-LIMIT

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ABSTRACT. Let $\omega(\cdot)$ denote the usual $\omega$-limit and in this paper we prove that, for given condition of the product map $f_1 \times \cdots \times f_m$ and

This result enriches the theory of

the form

$F(x_1, \ldots, x_m)$

where $\sigma$ is a permutation of the set of $\omega(F)$ is closed and we also show that $F$. These results solve open problems on topological dynamics of

AMS (MOS) Subject Classification

1. INTRODUCTION

Let $X$ be a compact metric space from $X$ into itself. Put $I := [0, 1]$ and $\varphi \in C(X)$ and $x \in X$ we consider $\omega$-limit set $\omega_\varphi(x)$ of the point $x$ in $\varphi$-orbit $\varphi_{\text{orb}}(x)$. Finally,

is the $\omega$-limit set of the map $\varphi$.

Consider now $f_1, \ldots, f_m \in C(X)$

(1.1)

$\omega(f_1 \times \cdots \times f_m)$

Clearly in some particular cases this equation is satisfied. On the other hand

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