Kaluza-Klein and Extra-Dimensions Field Theory

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January 29, 2019
The Standard Model

Interactions = Forces


General Relativity

Gravity = Curvature

https://www.space.com/17661-theory-general-relativity.html
Motivation to Find Unified Theory of Gravity

Introduction to Brief (Very Brief) Kaluza-Klein Theory

Theodor Kaluza

Oskar Klein

© Left: University of Göttingen, Right: Stanley Deser.
https://physociety.wordpress.com/2014/02/12/extra-dimension-for-dummies/
4th-Dimension vs 5th-Dimension (Metric Tensor)

- 4th-dimensional metric tensor

\[
g^{(4)}_{\mu\nu} = \begin{pmatrix}
g_{11} & g_{12} & g_{13} & g_{14} \\
g_{21} & g_{22} & g_{23} & g_{24} \\
g_{31} & g_{32} & g_{33} & g_{34} \\
g_{41} & g_{42} & g_{43} & g_{44}
\end{pmatrix}
\] (1)

- 5th-dimensional metric tensor

\[
g^{(5)}_{AB} = \begin{pmatrix}
g^{(4)}_{\mu\nu} - a^2 A_\mu A_\nu & -a^2 A_\mu \\
-a^2 A_\nu & -a^2
\end{pmatrix}
\] (2)
4th-Dimension vs 5th-Dimension (Curvature)

- 4th-dimensional curvature tensor

\[ R_{bd}^{(4)} = \partial_a \Gamma_{bd}^a - \partial_d \Gamma_{ba}^a + \Gamma_{bd}^e \Gamma_{ea}^a - \Gamma_{ba}^e \Gamma_{ed}^a \]  \hspace{1cm} (3)

- 5th-dimensional curvature tensor

\[ R^{(5)} = R^{(4)} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \]  \hspace{1cm} (4)

where \( F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \)
4th-Dimension vs 5th-Dimension (Summary)

\[ g^{(5)}_{\mu\nu} = \begin{pmatrix} g^{(4)}_{\mu\nu} - a^2 A_\mu A_\nu & -a^2 A_\mu \\ -a^2 A_\nu & -a^2 \end{pmatrix} \]

\[ R^{(5)} = R^{(4)} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \]
Variational Method

Classical Mechanics

\[ \delta S = \delta \int dx \, dt \, L \]
\[ = \int dx \, dt \left( \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} \right) \]

Equation of Motion for One Particle

Classical Electromagnetic and Gravitational Field

\[ \delta S = \delta \int dx^4 \, \left( -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} \right) \]
\[ = \int dx^4 \left( \partial_\mu F^{\mu \nu} \right) \]
\[ \delta S = \delta \int dx^4 \, R \]
\[ = \int dx^4 \left( R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R \right) \]

Maxwell's and Einstein's Equation
Unification of Electromagnetism and Gravity

The 5th-dimension gravity contains both Einstein’s gravity and Maxwell’s electromagnetism.

\[ S = \int dx^5 \sqrt{-g^{(5)}} R^{(5)} \]
\[ = \alpha \int dx^4 \sqrt{-g} R^{(4)} - \int dx^4 \sqrt{-g} \frac{1}{4} F_{\mu \nu} F^{\mu \nu} \]

They obtain non-interacting unified theory.
Results

Quantum Interpretation, Compactification-String Theory
quantum interpretation (extra dimension)

de Broglie relation for momentum

\[ p^{(5)} = \frac{h}{\lambda^{(5)}} \Rightarrow \lambda^{(5)} \sim \frac{h G^{1/2}}{cq} \sim 10^{-30} cm \]

The 5th dimension is a tiny circle of radius \( \sim 10^{-30} cm \) which far smaller than what experimentalist can see
Compactification

$M \times C$

Kaluza-Klein

$M$
Conclusion

- The unification of gravity and gauge field is not able to comprehend by traditional quantum field theory or general relativity.

- The extra dimension are one of the first ideas which unify gravitational and electromagnetic interaction.

- The Kaluza-Klein theory is non-interacting unified theory.

- If the fifth dimension exists it size is in -30 order of magnitude.

- The compactification of Kaluza-Klein theory is the first step of the string theory.


Kaluza-Klein Theory: Seminar 4 Anze Zaloznik. April 2012

Kaluza-Klein for Kids, William O. Straub, June 2014
Acknowledgements

- Udom Robkob, Ph.D.  (Advisor)
- Suraphong Yuma, Ph.D.
- Petchara Pattarakijwanich, Ph.D.
- Sutthipong Noisagool, Ph.D.